Elastic Wave Propagation in Materials

Introduction

The concept of elasticity is an approximation to the low-strain behavior of real materials. An ideal elastic material deforms in proportion to the applied load and recovers instantaneously both to its original dimensions and its original state (no damage) when the load is removed. So an ideal "elastic wave" is a mechanical disturbance that propagates through a material causing oscillations of the particles of that material about their equilibrium positions but no other change. Real materials differ from this ideal in a number of ways but it is worth mentioning "dissipation" mechanisms right at the outset. These cause attenuation of elastic waves with distance traveled, the rate of attenuation usually depending on frequency.

1. Basics of Elastic Wave Propagation in Bulk Materials

The physics contained even within the ideal elastic wave equation for the completely general (anisotropic) case

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \lambda_{iklm} \frac{\partial^2 u_m}{\partial x_k \partial x_l} \tag{1}$$

(where ρ is the density, u_i are the components of the displacement vectors, λ_{iklm} is the elastic modulus tensor, and the summation convention is followed) is often as complex and surprising as that contained in Maxwell's equations of electromagnetism.

Very rarely does anyone need to solve Eqn. (1) in its full generality, as symmetry leads to most of the elements being either zero, equal, or otherwise algebraically related. For instance, if the material has (i) the same mechanical properties in all directions (isotropic) and (ii) the microstructure can be ignored (small compared to the wavelength of the wave), then Eqn. (1) simplifies greatly to

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \mu \frac{\partial^2 u_i}{\partial x_k^2} + \left(K + \frac{4}{3}\mu\right) \frac{\partial^2 u_l}{\partial x_i \partial x_l} \tag{2}$$

where K is the bulk modulus and μ is the shear modulus.

Two different elastic waves, one dilatational (sometimes called longitudinal or compressional) and the other shear (sometimes called distortional or transverse), can thus now be identified which propagate through an unbounded isotropic medium. They travel with different velocities given by

$$c_1 = \sqrt{\frac{K + (4/3)\,\mu}{\rho}}, \quad c_2 = \sqrt{\frac{\mu}{\rho}} \tag{3}$$

Evidently $c_1 > c_2$, so "dilatational waves" arrive before "shear waves." In "seismology," the first are termed P waves (for primary) and the second S waves (for secondary). Situations where these waves are important include "earthquake detection" and "ultrasonic investigations" of materials and structures.

For more information, see the books by Kolsky (1953), Graff (1975), and Royer and Dieulesaint (2000).

2. The Effects of Boundaries

All finite objects have boundaries (surfaces). These strongly modify the behavior of elastic waves within the bulk and also result in new wave phenomena on or close to the surfaces.

One of the effects of boundaries on waves we are familiar with from our everyday experience of light is reflection and refraction. These effects arise from the various conditions that must be satisfied at the boundary (often assumed to be smooth but in reality rough). For elastic waves, these depend on the nature of the mechanical interface between the two materials in contact (see, for example, Comninou and Dundurs 1977). There are four basic possibilities:

(i) If a solid is in contact with a vacuum, the surface must be stress free.

(ii) If a solid is in contact with a fluid, the shear stress at the interface must be zero (fluids cannot support shear) but the normal component of the stress (i.e., the traction) must be continuous.

(iii) If two solids are firmly glued together, the particle motion at the surfaces of the two materials must be the same (else they would come apart); the stress at the surfaces of the two materials is given by the ratio of their elastic mechanical impedances ρc, where ρ is the density and c the elastic wave speed.

(iv) If there is a liquid or grease at the interface, again the tractions must be equal, but the particle motion can be different (surfaces are free to slip).

In general, bulk elastic waves of one of the two types identified above incident at an angle to the interface will reflect and refract as a combination of the two types (mode conversion). The sines of the angles involved are simply related to the wave speeds.

Due to the proximity of the boundary, rods, for example, act as "mechanical waveguides," and are used as such in applications such as "Hopkinson bars" for (i) high strain rate testing and (ii) measurement of blast waves. Another effect of the boundary is that elastic waves of different frequencies travel at different speeds, that is to say that rods are strongly dispersive. This results in distortion of elastic wave signals sent down by them. A full "three-dimensional" analysis is mathematically very complex (see, for example, the classic paper by Davies (1948)) but has to be performed if the wave that entered the rod is to be faithfully reconstructed. This is not usually necessary in high strain rate materials testing.

Elastic waves on surfaces are most commonly seen in everyday life on water. But similar phenomena also occur (usually unseen) at the surfaces of solids. Waves on solid surfaces can be of similar amplitude to waves on water as a result of earthquakes, large explosions (due to, for example, nuclear/hydrogen bombs or kilotons of high explosive), or meteorite impact, but in materials applications they are usually chosen to be of low amplitude and high frequency. High-amplitude elastic waves can be visualized in materials by their interaction with light (photoelastic effect), by the pattern of cracks they initiate, and by the effects they have on crack propagation.

If the material at the surface is the same as the bulk (no layers) the surface wave is termed as "Rayleigh wave" (see Fig. 1). It can be thought of as a combination of a longitudinal and a shear wave, each of which decays exponentially with depth but at different rates (see Fig. 1). The motion of particles on a surface over which Rayleigh waves are propagating is two-dimensional rather than one. At large distances from a seismic event, the Rayleigh wave is often the easiest to detect since it decays in two dimensions whereas bulk elastic waves decay in three (for a schematic representation of this, see Fig. 1).

However, many surface wave problems of much practical interest involve "layered media." Examples include "geological prospecting" in areas where the rocks have been laid down by sedimentary processes, or raindrop impact on the coated forward-facing surfaces of aircraft. Various types of surface waves have been described and are named after those who analyzed them. For example, "Love waves" occur when a substrate has a single layer; "Lamb waves" occur in plates whose thickness is comparable to (or less than) their wavelength (thick plates can have two independent Rayleigh waves); and "Stoneley waves" propagate along the interface between two elastic media. In many geological problems, elastic waves cannot be analyzed algebraically as the properties of rocks vary with depth and position.

For more information, see the books by Biryukov *et al.* (1995) and Royer and Dieulesaint (2000).

3. Applications of Elastic Waves in Materials Science

Three increasingly important uses of elastic waves (ultrasound) in materials science are: (i) measurement of dynamic moduli, (ii) nondestructive testing (NDT) to detect cracks and other defects, and (iii) medical imaging. Uses (ii) and (iii) are evidently strongly connected to each other.

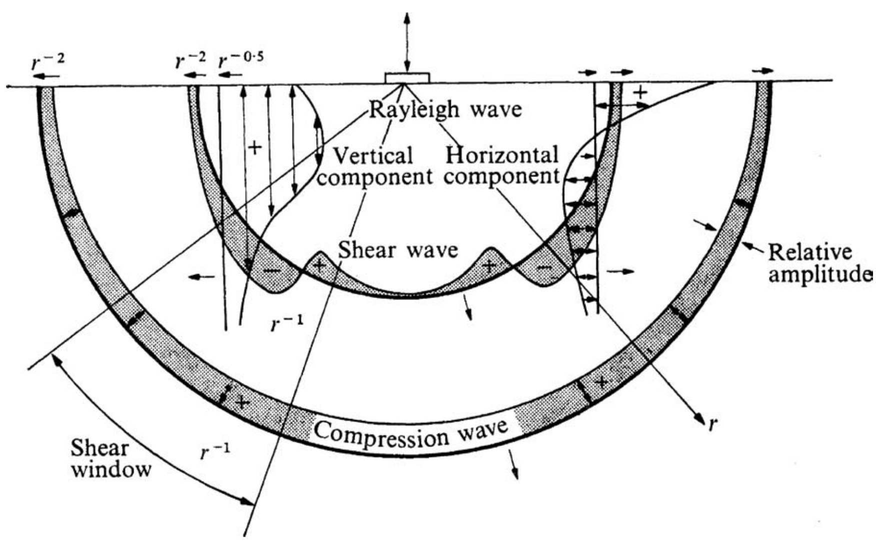

Figure 1
Schematic diagram of the excitation of compression (dilatational, longitudinal), shear (distortional, transverse), and Rayleigh waves (from Woods 1968). In this diagram, r is the distance in any direction in the solid from the source. The power-law dependence of the rate of decay of the three waves with r is given along the topmost line to the left (reproduced by permission of Graff 1975; © Dover publications).

It should be emphasized that ultrasonic techniques are the only viable methods of measuring elastic properties of materials at high strain rates. Other methods of dynamic testing (such as split Hopkinson bars) only become accurate after the specimen has yielded due to the time taken for stress equilibrium to be established in the specimen and hence measure plastic (post-yield) properties only.

Ultrasonic imaging techniques, making use of arrays or mechanically scanning transducers, are used in both NDT and medical applications. In the latter, there is also a requirement for techniques such as real-time imaging (framing rates of at least 5 per second) and Doppler effect detection of blood flow (both rate and direction), which are not generally required in NDT.

Recently, "harmonic imaging" has proved to be useful in medical applications and this technique can also be used in NDT. Harmonic imaging is possible when the material responds to ultrasonic waves in a nonlinear manner generating harmonics of the applied frequency. If the receiver is tuned to one or more of these other frequencies, greater sensitivity to variations in material properties results.

Irrespective of whether many transducers in an array or a single mechanically scanning transducer are used, good quality images of the surface and the interior of a test sample can be obtained using ultrasound. A lens design for such a "scanning acoustic microscope" (SAM) is shown schematically in Fig. 2. The interpretation of the contrast seen in the images obtained requires a thorough understanding of the theory of acoustic waves in solids and on their surfaces. Images with submicron resolution have been obtained. An example of an SAM image is shown in Fig 3. Nanometer resolution has been obtained by combining ultrasonic acoustic techniques with atomic force microscopy, the technique being termed "ultrasonic force microscopy" (see, for example, Dinelli *et al*. 2000).

For further details, see the books by Krautkrämer and Krautkrämer (1990), McIntire (1991), Briggs (1992), and Royer and Dieulesaint (2000).

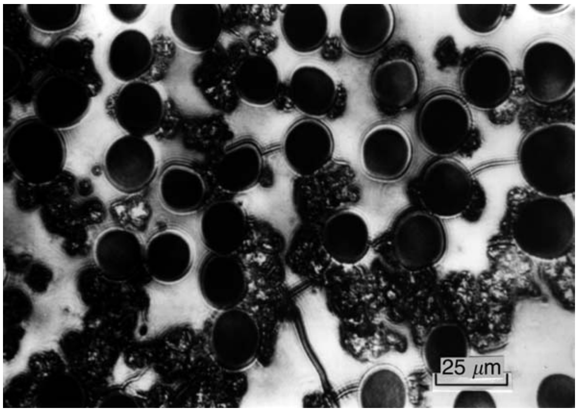

Figure 3
An SAM of a glass matrix, silicon carbide fiber composite. Figure courtesy of Professor Andrew Briggs.

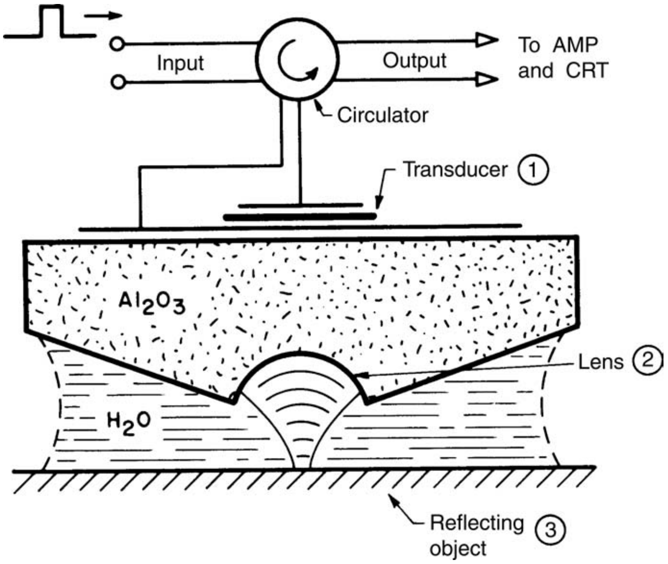

Figure 2
Schematic cross-sectional diagram of lens system for a scanning acoustic microscope (reproduced by permission of *Physical Acoustics*., 1979, Vol. 14, pp. 1–92; © Academic Press).

4. *Nonlinear Effects*

Bell pointed out in his survey of the history of mechanics (Bell 1973) that several contemporaries of Hooke criticized him for assuming a linear relationship between force and extension was generally true. Many materials were known for which this was not so. However, much analysis since then of small strain elastic problems has assumed linearity, although there is evidence that this is not so even for vanishingly small strains (Kochegarov 1999). So, for example, Eqns. (1) and (2) with which this review began, although algebraically complex, are still linear equations.

Nonlinear effects have their origin and basis in the anharmonic interatomic potential. A familiar manifestation of this "anharmonic potential" is the thermal expansion of solids (see, for example, Tabor 1991).

Less familiar are the following:

(i) The stiffness of matter depends on the hydrostatic pressure applied. Thus, the frequency of torsional oscillation of a rod (Birch 1937) and the elastic wave velocity of materials (Hughes and Kelly 1953) are both functions of pressure (see, for example, Fig. 4). There are clear implications for the interpretation of seismic waves traveling through the Earth and other planetary-sized bodies (Duffy and Ahrens 1992). If an anisotropic stress is applied to a material (e.g., a simple tension), ultrasonic waves are found to travel at different speeds depending on whether they are propagating parallel or perpendicular to the applied stress (e.g., Toupin and Bernstein 1961). This "acoustoelastic" effect can be used to measure the so-called third order elastic constants (Hughes and Kelly 1953). There is some possible confusion in terminology here (Smith 1963) because the third order elastic constants determine the magnitude of the second-order elasticity effects. The reason is that if you write down the general expression for the energy stored in a strained lattice $\Phi(\varepsilon)$, it has the following form (Toupin and Bernstein 1961):

$$\Phi(\varepsilon) = \frac{1}{2}\lambda_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + \frac{1}{6}\lambda_{ijklm}\varepsilon_{ij}\varepsilon_{kl}\varepsilon_{mn} + \dots \quad (4)$$

The λ_{ijkl} are second-order coefficients in the polynomial expansion yet they are used in first-order (classic) elasticity theory. Similarly λ_{ijklm} are third-order coefficients used to describe "second-order" departures from linear theory.

(ii) Elastic waves in solids (including phonons) are found to scatter off each other which they would not do if linear elasticity theory were true (Bateman *et al.* 1961, Jones and Kobett 1963).

For further details, see the books by Truell *et al.* (1969), Auld (1973), Anderson (1995), and Royer and Dieulesaint (2000).

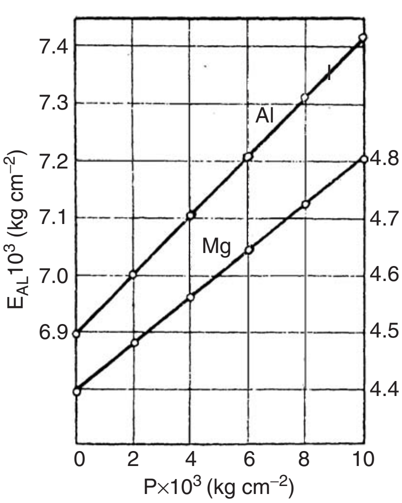

Figure 4
Graphs showing the effect of hydrostatic pressure on the ultrasonic Young's modulus of aluminum and magnesium (reproduced by permission of *Phys. Metals Metallog.*, 1961, **11**(3), 111–18).

5. *Shock Waves*

In the elastic waves considered above, the particles oscillate about their equilibrium position. However, in a shock the particles are given a motion in one direction only. Shocks can form in solids because the velocity with which elastic waves travel increases with pressure for the reasons discussed in Sect. 4. So if a violent event takes place at the surface of a body, such as an explosion or a high-velocity impact (few hundred m s^{-1} or above), the high-amplitude elastic waves that are generated "catch up" the low-amplitude waves forming a discontinuity in pressure, volume, particle velocity (and temperature) which propagates through the medium. Unlike in low-amplitude elastic waves where the particle velocity is linearly proportional to the amplitude, in a shock higher order terms in the pressure–volume relation need to be included. Often it is only necessary to include the first nonlinear term in the *p–V* relation; in other words a quadratic relation exists between pressure and particle velocity (the so-called Hugoniot relation). Every material has a unique experimentally determined Hugoniot (locus of possible shock states). However, it may be necessary to include higher-order terms for a number of reasons, e.g., the shock pressure is very high, the material is porous, and low-order phase changes occur.

The interface particle velocity and pressure generated when two materials collide at a known velocity is calculated using their Hugoniot curves by the so-called "crossed Hugoniot" method (see, for example, Fig. 5). The reason the Hugoniots are crossed

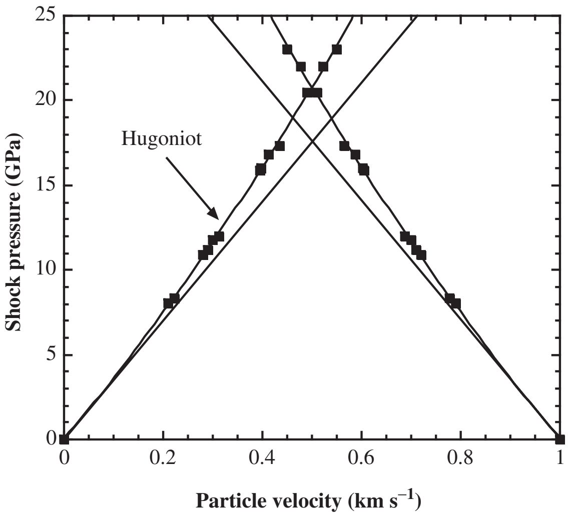

Figure 5
Graph showing the pressure calculated for the impact of copper on copper at 1 km s^{-1} using the crossed Hugoniot technique as compared with that calculated assuming linear elasticity.

is because at impact the impactor is decelerated and the target is accelerated. This is represented diagrammatically by the Hugoniot of the impactor starting at the impact velocity and rising to the left (i.e., the pressure rises as its velocity falls). In the target, however, the particle velocity rises as the pressure rises and this is represented by the Hugoniot of the target starting at its initial velocity (usually zero) and rising to the right. The point where the Hugoniots cross is the solution to the problem. This is because the pressure must be the same either side of the interface in both bodies else there would be a net force tending to accelerate one material away from the other. Similarly, the particle velocity must be the same or again the interface would open up. For the symmetric case given, the interface particle velocity is half the impact velocity, the same result as would be obtained using linear elasticity. Note, however, that considerable errors result if low-pressure elastic properties of materials are used to calculate "shock pressures." For example, in the situation shown in Fig. 5, the impact pressure that would be calculated using linear elasticity (given by $0.5\rho cV$) is 17.6 GPa as compared to 21 GPa calculated using the experimentally determined Hugoniot curves. The discrepancy will be worse at higher impact velocities and for materials with more nonlinear Hugoniots. Linear elasticity also gives the wrong answer for the interface velocity if dissimilar materials are in collision.

Most shock studies are carried out in one-dimensional strain either by plate impact loading or by the use of explosive lenses. This is largely because of the difficulties associated with making measurements under shock conditions if the material strains are in more than one dimension (Gran and Seaman 1997). However, the performance of many materials of interest under more complex dynamic conditions (such as may be generated in a high-velocity impact) can be related to their dynamic shear strength (Rosenberg *et al*. 1990). This can be measured in one-dimensional strain under shock conditions (Rosenberg *et al*. 1987).

Shock structures and phenomena are made more complicated and interesting by the elastic–plastic transition, material anisotropy, phase changes, and release of chemical energy (in explosives, for example).

For more information, see the books by Asay and Shahinpoor (1993), Graham (1993), Drumheller (1998), and Zukas and Walters (1998).

6. *Use of Elastic Waves in Geology and Mining*

As the dimensions of geological objects may be many orders of magnitude larger than typical laboratory specimens, elastic wave sources of high energy are usually required to obtain information about them. For this reason, the structure of solid planetary bodies, such as the Earth and Moon, was historically largely determined by examination of the propagation of waves generated by earthquakes, nuclear explosions, and artificial or meteoritic impact (Cook 1980).

As mentioned earlier, Eqns. (1) and (2) are true only so long as the elastic parameters λ do not vary within the region of interest. For planetary-sized bodies, these elastic parameters vary with depth and lateral position so that the seismic wave equations are much more complex as they contain terms taking these variations into account (see, for example, Shearer 1999, Chapter 3). Finite-difference codes are routinely used by geophysicists to produce synthetic seismograms of full "wave field propagation" through laterally inhomogeneous media.

As mentioned earlier, waves of different frequencies attenuate at different rates in real media. Thus, geophysicists have found that for the Earth elastic waves of frequency:

- 3–10 kHz propagate up to a few tens of meters;
- 200 Hz propagate up to a few hundred meters;
- 50 Hz propagate up to a few kilometers;
- 10 Hz propagate up to a few tens of kilometers;
- 1 Hz propagate up to a few hundred kilometers.

So the source must be tailored to the application. Generally, nowadays nonexplosive sources are used, for example, piezoelectric crystals for kHz generation under water or vibrators (20 tonne trucks) for land seismic studies. Chemical explosions are used to generate sound pulses for examination of the pattern of rocks beneath the surface of the Earth to a depth of a few tens of kilometers. However, for exploration under water this method has largely been replaced by the use of high-pressure gas guns to generate sound pulses via bubble formation and collapse.

In peacetime, the vast majority of explosives manufactured are used in quarrying and mining. The shape and duration of the pressure pulse needed for such applications are normally different to that required by the military. Rocks must simply be broken up into manageable pieces (from a few kilograms to a few tonnes) and must not normally be pulverized to dust. This means the pressure pulse must build up more slowly and be of longer duration and lower amplitude compared to most military applications. The aim is to produce a network of cracks so that the rock is broken into pieces of a size suitable for transport to a processing plant (see, for example, Field and Ladegaard-Pedersen 1971).

For more information, see the books by Persson *et al.* (1994), Petrosyan (1994), Anderson (1995), Borovikov and Vanyagin (1995), Baranov *et al.* (1996), Bhandari (1997), Hustrulid (1999), Shearer (1999), and Poirier (2000).

7. *Wave Phenomena in Ballistic Impact*

Ballistic impact on the battlefield at present normally takes place in two main velocity ranges: 1–3 $\mathrm{km\,s^{-1}}$ (bullets, armour penetrators) or at 6–8 $\mathrm{km\,s^{-1}}$ (shaped charges). In the lower velocity range (1–2 $\mathrm{km\,s^{-1}}$), the mechanical strength of the target and missile are important. Above $\sim$2–3 $\mathrm{km\,s^{-1}}$ (the sort of velocities achievable by explosively formed projectiles), the shock pressures are such that the mechanical strength of the target can be largely ignored. In other words, the material behaves temporarily as if it were a liquid, i.e., hydrodynamically. Above $\sim$6 $\mathrm{km\,s^{-1}}$ (velocities typical of shaped charges, space debris, and meteorites), the energy density of the shock generated is comparable to the cohesive energy of the solid. This results in the vaporization of the material of the target and impacting body so that a mechanical explosion occurs.

As the schematic diagram (Fig. 6) illustrates, wave phenomena in the general case of oblique long-rod ballistic impact are very complex. Analytical solutions exist for the relatively simple case of normal impact, but oblique impact can only be modeled numerically or studied experimentally (Zukas 1990, Wilkins 1999). However, the deceleration of the rigid (nondeforming) rear section of the rod can be estimated using an analysis similar to that of Taylor (1948) for rod impact on a rigid surface. The deceleration is caused by the reflection of the longitudinal elastic wave from the rear surface. This wave travels up and down the length of the rod producing an incremental change in the velocity ΔV each time of $-2Y/\rho c$.

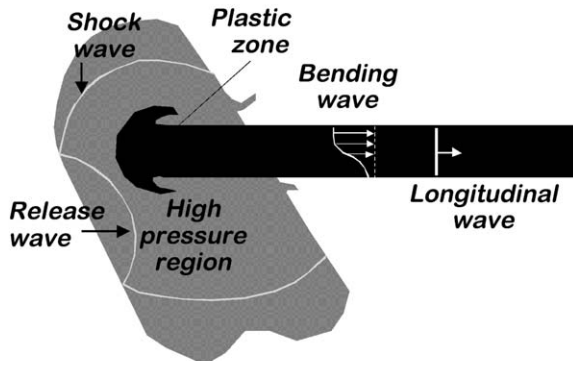

Figure 6
Schematic diagram of ballistic impact of a rod on a target (reproduced by permission of *High Velocity Impact Dynamics*, 1990; ©Wiley).

For further information, see the books by Zukas (1990), Carleone (1993), Lloyd (1998), and Wilkins (1999).

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